1. PHD PROJECT DESCRIPTION (4000 characters max., including the aims and work plan)

Project title:

Derived equivalences for the derived discrete algebras

1.1. Project goals

The aim of the project is to describe the equivalences for the derived discrete algebras. In particular, our aim is to prove that the derived equivalences for the derived discrete algebras are standard. We plan to use this result in order to describe the derived Picard group for the derived discrete algebras.

1.2. Outline

Vossieck [V] defined in 2001 a class of the derived discrete algebras, where an algebra A is said to be derived discrete if, for each collection (h_i) of nonnegative integers, there is only finitely many isomorphism classes of objects X in the derived category $\mathrm{D}^b(\mathrm{mod}\ A)$ such that (h_i) is the cohomology dimension vector of X. Vossieck proved that an indecomposable algebra A over an algebraically closed field is derived discrete if and only if either A is derived equivalent to a hereditary algebra of Dynkin type or A is a one-cycle gentle algebra not satisfying the clock condition.

A famous result due to Rickard [R1, R2] says that if algebras A and B are derived equivalent, then there exists a standard equivalence between $\mathrm{D}^b(\mathrm{mod}\ A)$ and $\mathrm{D}^b(\mathrm{mod}\ B)$, i.e. an equivalence given by the derived tensor functor by a two-sided titling complex. This is a well-known open question if, for a given algebra A, every derived equivalence starting at $\mathrm{D}^b(\mathrm{mod}\ A)$ is a standard one. Chen and Zhang [CZ] gave an affirmative answer to the problem in the case of the derived discrete algebras of finite global dimension. This is however not known if their result extends to the derived discrete algebras of infinite global dimension.

The main aim of this project is to confirm the conjecture in the case of the derived discrete algebras of infinite global dimension.

1.3. Work plan

First year will be devoted to expanding knowledge of the PhD candidate on representation theory of finite dimensional algebras, with a particular emphasis on their derived categories.

Our first aim will be to prove that the derived equivalences of the derived discrete algebras of infinite global dimension are standard. In order to achieve this aim we plan to to use:

- A description of the derived categories for these algebras obtained in [BGS] and [BPP].
- A criterion of Chen and Ye [CY].

Next we plan to study the derived Picard group for this class of algebras.

1.4. Literature

[BGS] G. Bobiński, Ch. Geiß, and A. Skowroński, Classification of discrete derived categories, Cent. Eur. J. Math. 2 (2004), 19–49.

[BPP] N. Broomhead, D. Pauksztello, and D. Ploog, Discrete derived categories I: homomorphisms, autoequivalences and t-structures, Math. Z. 285 (2017), 39–89.

[CY] X.-W. Chen and Y. Ye, The D-standard and K-standard categories, Adv. Math. 333 (2018), 159–193.

[CZ] X.-W. Chen and Ch. Zhang, The derived-discrete algebras and standard equivalences, J. Algebra 525 (2019), 259– 283.

[R1] J. Rickard, Morita theory for derived categories, J. London Math. Soc. (2) 39 (1989), 436–456.

[R2] J. Rickard, Derived equivalences as derived functors, J. London Math. Soc. (2) 43 (1991), 37–48.

[V] D. Vossieck, The algebras with discrete derived category, J. Algebra 243 (2001), 168–176.

1.5. Required initial knowledge and skills of the PhD candidate

Knowledge of algebra and homological algebra is required. Basic training in representation theory of finite dimensional algebras would be an advantage.

1.6. Expected development of the PhD candidate's knowledge and skills

After completing the project the PhD candidate:

- will have a deep knowledge of representation theory of finite dimensional algebras;
- will be able to find and solve research problems in mathematics.