1. PHD PROJECT DESCRIPTION (4000 characters max., including the aims and work plan)

Project title:

Detection and quantification of multipartite entanglement

1.1. Project goals

The project goal will be to look for generalisations of Computable Cross-Norm Criterion and Covariance Matrix Criterion to multipartire case. The student will take part of research started in publications arXiv:2001.08258, arXiv:2002.00646. We are interested in related families of linear and non-linear entanglement witnesses related to the criteria we will find as well as possible tomographic shemes and protocols related to them. Such new tomographic schemes and quantum-information protocols will give rise to some physicaly-motivated measures of entanglement in a given multipartite state.

1.2. Outline

In Quantum mechanics states are represented by positive, trace-class operators, having trace normalized to identity, acting on a complex Hilberts space, known as *density operators* (or density matrix, when the basis is fixed). Hilbert space of a composite system is a tensor product of the Hilbert spaces of the subsystems and a product state - when there is no correlations between subsystems - is given by a tensor product of the density operators of subsystems:

$$\rho = \rho^{(A)} \otimes \rho^{(B)}$$

If it is possible to prepare such states locally (due to the lack of correlations). We can produce a correlated state mixing the product states (for example one can add in the state preparation protocol tossing a dice to choose a product state to prepare):

$$\rho = \sum_i p_i \rho_i^{(A)} \otimes \rho_i^{(B)}$$

States of the above form are called *separable states* and form a proper subset of the set of states - not all quantum states using such protocols. Quantum mechanics, being a non-kolmogorov probability theory, admits correlations between subsystems of a bigger system, which cannot be explained by classical theory. We call states with such correlations *entangled states*. Entanglement first have been treated as a proof of non-completeness of quantum mechanics, but after they have been shown experimentally, physicists accepted the quantum mechanics as it is, despite its counter-intuitivity. Now there exist quantum-information protocols, like quantum cryptography and quantum teleportation which uses non-classical properties of quantum correlations. Quantum entanglement become a resource for these protocols. Problems of entanglement detection, its dynamics prediction or how to preserve it become of growing importance from the point of view of possible applications.

First of the mentioned problems - detecting entanglement between two (in the simplest case) subsystems of a given system is hard even if the state of the system is explicitly given as a density matrix. Because entanglement is the key resource for the novel information protocols, various necessary and sufficient conditions for the state to be entangled has been developed. It is worth to remind here, that in the real-life scenarios a density matrix is not given - one can reconstruct it in the process of *quantum tomography*, which requires a lot of state-destructing measurements to perform (the number of measurements grows like n^2 with the dimension of the Hilbert space and like ϵ^{-2} with the desired accuracy). Hence methods of detecting entanglement without full tomography and optimising the number of measurements are of great interest.

The theory of convex sets / convex cones shows, that for any entangled state there exists an observable (measurable quantity, analogon of measurable function in the classical probability theory) positive on all separable states but negative in the given entangled state. We call such an observable an *entanglement witness*. To detect entanglement of a $n \times m$ system one has to measure one global observable of the

system, not $(n \cdot m)^2 - 1$ of them which would be necessary for the full system tomography. Measuring one appropriately chosen entanglement witness we can detect entanglement of a given state without knowing its density matrix.

Except for the lowest dimensions of the subsystems, the classification of entanglement witnesses is not known (otherwise the problem of entanglement would be solved). The famous Jamiołkowski isomorphism relate entanglement witnesses between positive, but not completely positive maps between matrix algebras (or C^* algebras in general) and in this way separability problem is related to another mathematical problem - classification of positive maps.

Another important entanglement criterion is so called *realignment criterion*, *cross-norm criterion* or *CCNR criterion* (and its variuos generalisations).

1.3. Work plan

Under my supervision, a candidate will be expected to get familiar during the PhD studies with:

1. **Theory of positive maps between matrix algebras, especially for detecting quantum entanglement.**

2. **The theory of entanglement witnesses, Bell inequalities and detecting correlations in the distant-laboratories scenario.**

3. **Theory of quantum tomography and estimation, tomographic shemes: SIC POVMs, MUBs**

4. **Theory of multipartie entanglement.**

5. **Theory of Lie groups their actions, sets of orbits and their polynomial invariants.**

6. **Matrix norms, theory of multidimensional matrices and their invariants**

7. **Algorithms of Data Science for multidimensional data sets.**

8. **Methods of numerical verification of new hypotheses, linear and semidefinite programming**

1.4. Literature

1. **K. Chen and L.-A. Wu, Quantum Inf. Comput. \textbf{3}, 193 (2003)**

2. **O. Rudolph, Quantum Inf. Proccess. \textbf{4}, 219 (2005)**

3. K. Chen and L.-A. Wu, Phys. Rev. A \textbf{69}, 022312 (2004)

4. O. G\"uhne et al., Phys. Rev. A \textbf{74}, 010301(R) (2006)

5. C.-J. Zhang et al., Phys. Rev. A \textbf{76}, 012334 (2007)

6. C.-J. Zhang, Y.-S. Zhang, S. Zhang, and G.-C. Guo, Phys. Rev. A \textbf{77}, 060301(R) (2008)

7. O. G\"uhne, P. Hyllus, O. Gittsovich, and J. Eisert, Phys. Rev. Lett. \textbf{99}, 130504 (2007)

8. O. Gittsovich and O. Gühne, Phys. Rev. A \textbf{81}, 032333 (2010)

9. M. Li, S.-M. Fei, and Z.-X. Wang, J. Phys. A: Math. Theor. \textbf{41}, 202002 (2008)

10. P. Badzi\c{a}g, C. Brukner, W. Laskowski, T. Paterek, and M. Żukowski, Phys. Rev. Lett. \textbf{100}, 140403 (2008)

11. W. Laskowski, M. Markiewicz, T. Paterek, and M.Żukowski, Phys. Rev. A \textbf{84}, 062305

(2011)

12. J. D. Vicente, Quant. Inf. Comput. \textbf{7}, 624 (2007)

13. J. Shang, A. Asadian, H. Zhu, and O. G\"uhne, Phys. Rev. A \textbf{98}, 022309 (2018)

14. G. Sarbicki, G. Scala, and D. Chruściński, Phys. Rev. A 101, 012341 (2020)

15. G. Sarbicki, G. Scala, and D. Chruściński, arXiv:2002.00646

1.5. Required initial knowledge and skills of the PhD candidate

MSc in theoretical physics, mathematics or computer science

1.6. Expected development of the PhD candidate’s knowledge and skills

2. Theory of positive maps between matrix algebras, especially for detecting quantum entanglement.

3. The theory of entanglement witnesses, Bell inequalities and detecting correlations in the distant-laboratories scenario.

4. Theory of quantum tomography and estimation, tomographic shemes: SIC POVMs, MUBs

5. Theory of multipartie entanglement.

6. Theory of Lie groups their actions, sets of orbits and their polynomial invariants.

7. Matrix norms, theory of multidimensional matrices and their invariants

8. Algorithms of Data Science for multidimensional data sets.

9. Methods